QCD mass scale, proton mass and threshold heavy-quarkonium production

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Outline

- Mass scale
- QCD mass scale and ways to fix it
- Influences of the nucleon mass
- Origins of the nucleon mass
- Physics of the anomaly contribution
- Conclusion

Mass scale

- Mass scale is one of the important parameters in a theory.
- In the case of atoms, molecules, chemistry and biology, to a good approximation, there is a single mass scale that matters, the electron mass.

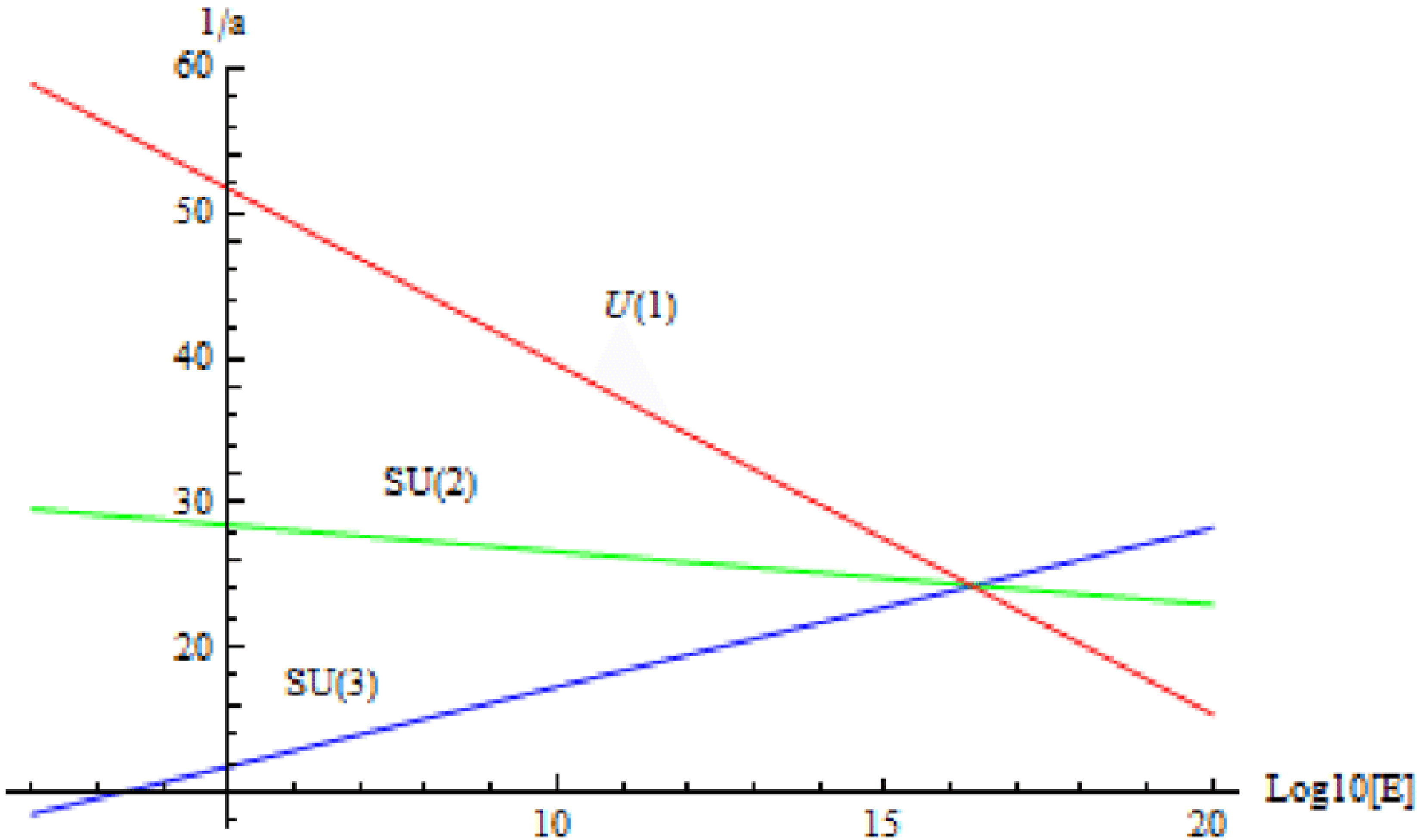
e.g. $E_n = -\frac{\alpha_{em}^2}{2n^2} m_e$

the coefficient depends on the specific dynamics.

Mass scales in QCD

- For QCD, there are two types of mass scales:
  - **Quark masses**, just like the electron mass in atomic physics, determined by Higgs mechanism
  - **QCD scale $\boldsymbol{\Lambda_{QCD}}$**, a parameter
- QCD scale Λ_{QCD} does not appear directly in the lagrangian.
  - Quantum anomaly or scale anomaly, in the massless limit, without UV divergences, the trace of EMT is zero.
  - However, due to UV divergences, the trace is now zero $T_{\mu}^{\mu} \neq 0$, proportional to QCD β-function
  - Scalar, generating a scale.

What decides the QCD scale?



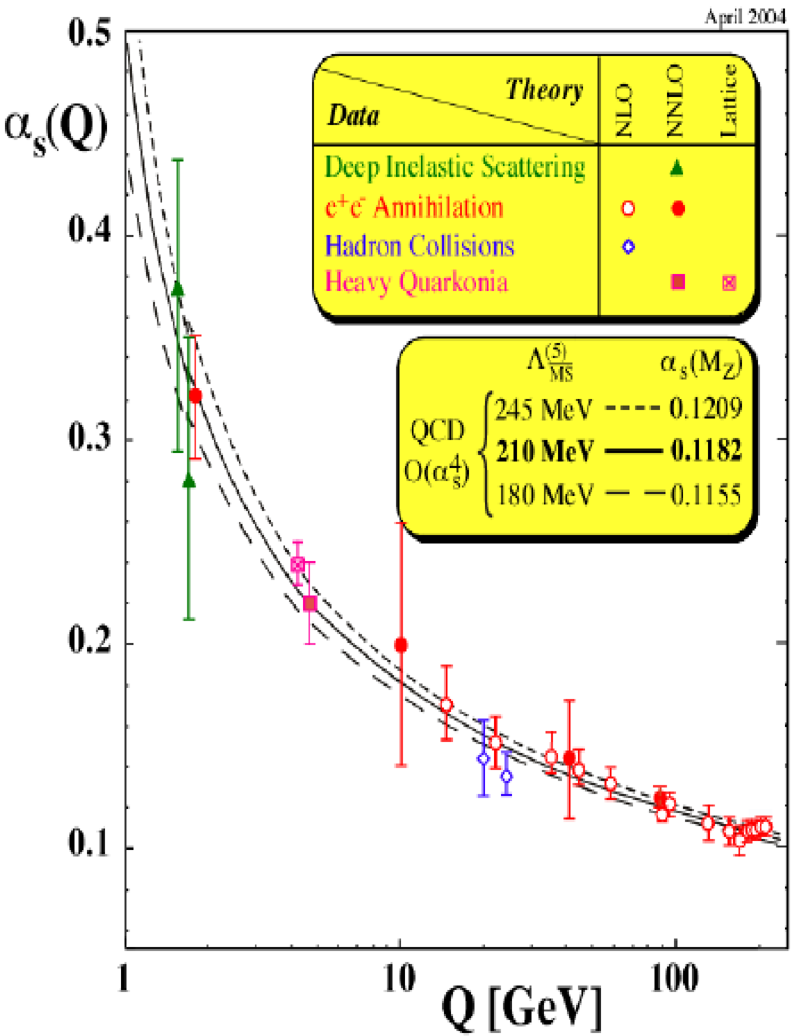
Fixing the QCD mass scale I:

- Trace anomaly

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^2$$

- Two ways to get the scale Λ_{QCD}

1. measuring the scale evolution coupling or the β-function



Fixing the QCD mass scale II

- Directly measure the matrix elements of the trace anomaly $\langle T_\mu^\mu \rangle$
- The matrix element in the vacuum state,

  gluon condensate: $\langle 0 | F^2 | 0 \rangle_{n.p.} \sim c\Lambda_{QCD}^4$

  - This has been extracted from QCD sum rules.
  - c is possibly related to QCD confinement, can be calculated from lattice QCD
  - Λ_{QCD} can be determined

Fixing the QCD mass scale III

- Matrix element in the nucleon

$$\langle p|F^2|p\rangle \sim f\Lambda_{QCD}$$

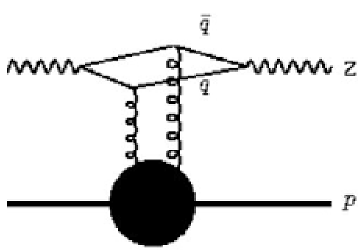
Therefore, a measurement of $\langle p|F^2|p\rangle$ is an alternative way to get the mass scale.

The coefficient f is non-perturbative, related possibly to color-confinement, and can be calculated in lattice QCD.

If $\langle p|F^2|p\rangle$ can be measured experimentally, it offers a new way to extract the QCD scale.

Measuring $\langle p|F^2|p\rangle$

- This is a measurement of color fields inside the nucleon.
- The best approach is to have a color dipole
- This naturally leads to the "probe" through electro-production of heavy quarkonium.
- Precision is the key!



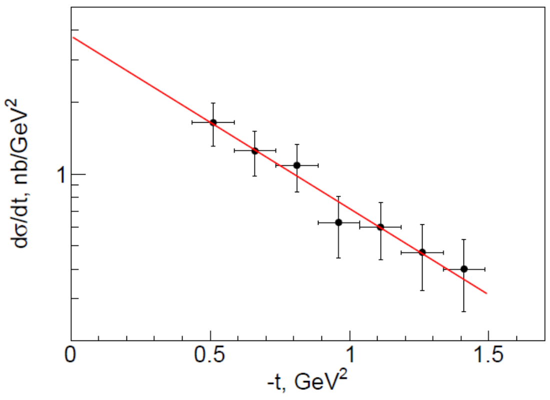


Fig. 1. The differential cross section of J/Ψ photo-production near threshold as measured by the GlueX Collaboration [22]. Only statistical uncertainties are shown.

1. QCD scale determines the size of the nucleon mass, which has enormous influences on the world.
2. Its content depends on the dynamics,
3. The anomaly matrix element again plays an important role

Roles of the nucleon mass

- Proton and neutron masses account for 5% of the energy of the Universe.
- The mass is a charge of gravity, and determines the galaxy and star formation dynamics.
- The mass determines the nuclear energy releases during fission and fusion, and hence the star evolution.
- Understanding the nucleon mass is an important mission of nuclear physics.

How do we understand the nucleon mass?

- Mass is energy!

  $M = E/c^2$

  to understand the mass, it is to understand the energy of QCD dynamics.
- QCD energy is determined by QCD Hamiltonian
- Where is the QCD dynamics?

  $$M = f\Lambda_{QCD}$$

Splitting the QCD energy source

- Splitting the energy

$$H_{\rm QCD} = H_q + H_m + H_g + H_a .$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ ← Quark energy

$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ ← Quark mass

$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2),$$ ← Gluon kinetic energy

$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2).$$ ← Trace anomaly

Is the anomaly contribution avoidable?

- Symmetry argument

  $H = \int T^{00}$

  T^{00} contains both traceless and traceful parts.

  traceless part and traceful parts are separately

  scale invariant.

Therefore, the scale-invariant nucleon mass is a sum of two scale invariant parts.

One can miss this in Dim Reg.

Metz et al. https://arxiv.org/abs/2006.1117

Relativistic “Virial theorem”

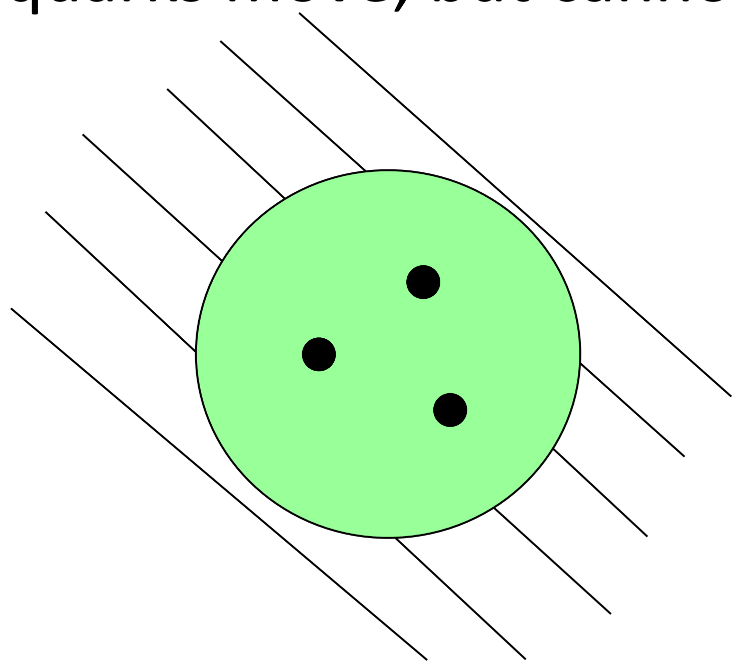
- Traceful part contributes ¼ of the nucleon mass.

 This results seems trivial in mathematics but perhaps very deep in physics.

- It is related to confinement through the action of the QCD vacuum?

Color Confinement---In a Bag!

- The quark confinement leads to that a quark in the nucleon must move in a small region of space.
- Therefore, a hadron looks like a bag inside which the quarks move, but cannot go to the outside.



The Mass of A bag, Along with 3 Quarks

- A free quark inside of the nucleon has a kinetic energy 1/R, according to the uncertainty principle.
- However, the free space of volume V has energy BV—you must pay for the bag!
- Therefore, the total energy is

$$M = \frac{3}{R} + \frac{4}{3}\pi R^3 B$$

- Minimizing with respect to R, one finds that the second term contributes 1/4 and M=4/R. And since R is about 1 fm, one gets about 900 MeV!

Anomaly vs. dark energy

- The anomaly is represented by the energy momentum tensor density $Bg^{\mu\nu}$ which is mathematically the same as the cosmological constant (CC) term $\Lambda g^{\mu\nu}$ introduced by Einstein.
- It is known today that the dark energy contributes about ¾ of the energy density of the Universe.