Comparison between Wiser π^- rates calculation and data from transversity and PVDIS experiments

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Abstract

The Wiser code has been used widely to estimate the pion background in electron scattering experiments. However, there has been little effort to understand how it was adapted to electron scattering. We hope to fill this gap in this writeup. We also provide comparison between Wiser calculation and data from the 6 GeV transversity and PVDIS experiments.

1 Introduction – The Wiser Code

Parametrization of hadron production in electron scattering (e, N) is a convenient tool for the design of nuclear physics experiments and the analysis of the resulting data. Parametrized cross sections can be used in radiative correction, calculating experimental background, and experimental feasibility design such as for $(e, e'N)$ coincidence measurements.

For experiments at (earlier) SLAC and at JLab, a widely used code is called Wiser, based on a parameterization of inclusive photoproduction of protons, kaons, and pions (for kaons and pions both charges were included) [1]. The Wiser code is based on the early SLAC data using: (1) a hydrogen target, (2) a bremsstrahlung photon beam made from the electron beam, with endpoints of 5, 7, 9, 11, 15, and 19 GeV, and (3) the SLAC 8 GeV/c spectrometer. The lowest momentum coverage for π^- is either 1 or 2 GeV/c, for K^- between 1 and 3 GeV/c, and for p or $\bar{p}$ between 1 and 4 GeV/c, depending on the beam energy. Because bremsstralung photon beam was used, a method called bremsstralung subtraction was used to extract the cross section for a monochromatic photon beam. The final results were given for the pion, kaon, proton and anti-proton in the form of [Eqs. (IV-A-2) and (IV-A-2) of [1]]:

$$E\frac{d^3\sigma}{dp'^3} \tag{1}$$

where E is the electron beam energy and p' the hadron momentum. The Fortran routine WISER_ALL_FIT returns the value of $E\frac{d^3\sigma}{dp'^3}/K$ with K a specific photon energy. Data from a deuterium target were available, but were not bremsstrahlung-subtracted and only integrated cross sections were included in Ref. [1].

Because of the specific kinematic regime of the data, it is expected that the Wiser code should work reasonably well for the multi-GeV beam energy at JLab. To obtain the hadron cross section for an electron beam, we essentially do an integral which is the inverse of Wiser's analysis, see Eq. (III-A-4) of [1]:

$$\frac{d\sigma}{dp'd\Omega} = \frac{p'^2}{E}\left(\frac{K_\gamma^{\rm total}}{E}\right)\int_{K_{\rm min}}^{E} E\frac{d^3\sigma}{dp'^3}\frac{\alpha(K,K_0)}{K}dK \tag{2}$$

where $K_{\rm min}$ is the minimum photon energy required for producing the hadron at the given p' and $\alpha(K, K_0)$ is a bremsstrahlung factor. The upper limit of the integral is the maximum bremsstralung photon energy K_0 and can be taken as the electron beam energy E. The fraction $K_\gamma^{\rm total}/E$ describes the total energy contained in the bremsstrahlung beam relative to the electron beam energy, which if multiplied by the running times gives EQ, the number of equivalent quanta in Ref. [1].

The number of photons as a function of the photon energy K for a given E falls roughly as $1/K$, and is generally written as

$$\frac{dn}{dK}(K,E) = b\frac{\alpha(K,K_0)}{K}dK \quad (3)$$

where α describes the deviation of the spectrum from a $1/K$ shape, and b is proven to be equal to EQ. The bremsstrahlung photon beam current can thus be written as

$$\frac{dn}{dtdK}(K,E) = \frac{K_\gamma^{\rm total}}{E}\frac{\alpha(K,K_0)}{K}dK\ . \quad (4)$$

The normalization is such that $\int_0^{K_0}\alpha(K,K_0)dK = K_0$ and to a good approximation we can take $\alpha(K,K_0) = 1$.

The integral is done in the code

WISER_ALL_SIG(E1,PTP,THP,RAD_LEN,ITYPE,TOTAL)

where E1 is the electron beam energy or max of bremsstralung spectra in MeV; PTP and THP are the momentum and the scattering angle of the outgoing hadron, in MeV and degrees respectively; RAD_LEN is the input radiation length in percent including both internal and external, ITYPE specifies the hadron type, and TOTAL is the output cross section $d\sigma/dE'd\Omega$ in nb/GeV-str. The calculationof $d\sigma/dE'd\Omega$ is done as

$$\frac{d\sigma}{dp'd\Omega} = \frac{p'^2}{E}\frac{\rm RAD\_LEN}{100.}\int_{K_{min}}^{E} E\frac{d^3\sigma}{dp'^3}\frac{1}{K}dK \quad (5)$$

where we can see that the approximation $\alpha(K,K_0) = 1$ was already taken and the input radiation length should describe the fractional energy that has been converted to bremsstrahlung photons, $K_\gamma^{\rm total}/E$.

We refer to WISER_ALL_SIG as the *Wiser* code hereafter.

1.1 The Equivalent Photon Radiator

We denote t as the total radiation length input to the *Wiser* code, RAD_LEN. It must include both internal and external radiators. The bremsstrahlung conversion of electron beam passing through material is given by Eq. (32.30) and (32.31) of Ref. [4], which for small thickness (fraction of the radiation length X_0) is given by

$$\frac{d\sigma}{dK} = \frac{A}{X_0 N_A K}\left(\frac{4}{3}-\frac{4}{3}y+y^2\right) \quad (6)$$

with A the atomic number of the target and N_A the Avogadro number. This can be integrated to give the total number of photons

$$N_\gamma = \frac{t_{\rm material}}{X_0}\left[\frac{4}{3}\ln\left(\frac{k_{\rm max}}{k_{\rm min}}\right) - \frac{4(k_{\rm max}-k_{\rm min})}{3E} + \frac{(k_{\rm max}^2-k_{\rm min}^2)}{2E^2}\right) \quad (7)$$

where $y = K/E$. To simplifies the calculation, we note that $0 < y < 1$ and $\left(\frac{4}{3} - \frac{4}{3}y + y^2\right) < \frac{4}{3}$, also $E_\gamma < E$ and $E_\gamma^{\rm total} < N_\gamma E$, therefore the effective material thickness that can be used as input to the *Wiser* code is

$$t_{\rm ext} = \frac{4}{3}\frac{t_{\rm material}}{X_0}. \quad (8)$$

The internal radiation $t_{\rm int}$ is from the electron (de)acceleration in the presense of the target's electromagnetic fields. There are at least two ways of calculating $t_{\rm int}$. In Mo&Tsai's treatment of (e, e') reaction [2] (first equation, un-numbered), internal bremsstrahlung was thought to have roughly the same effect as that given by two external radiators with one placed before and one after the scattering, each of thickness

$$t_{\rm i,f} = \frac{3}{4}\frac{\alpha}{\pi}\left[\ln\left(-q^2/m_e^2\right) - 1\right] , \quad (9)$$

in units of radiation lengths. Here m_e is the electron mass and q^2 is the 4-momentum transfer (squared) from the electron to the target. Note that Eq. (9) already took into account the factor $4/3$ of Eq. (8), thus the input to the *Wiser* code, if using Mo&Tsai, should be

$$\frac{4}{3}t_{\rm i,f} = \frac{\alpha}{\pi}\left[\ln\left(Q^2/m_e^2\right) - 1\right] . \quad (10)$$

Here, only the before-radiator should be used. (The before-radiator affect all electrons in the beam, while the after-raditor only affect electrons that are scattered and the resulting bremstrahlung is suppressed by several orders of magnitude). Therefore

$$t_{\rm int,Mo\&Tsai} = \frac{\alpha}{\pi}\left[\ln\left(Q^2/m_e^2\right) - 1\right] . \quad (11)$$

Eq. (10) can in principle be used directly as the internal radiation part of the RAD_LEN, input to the *Wiser* code. However, the Q^2 is defined for the (e, e') kinematics and is missing for inclusive hadron production. If the final state proton or meson are produced from nucleon resonance decay, it's possible to calculate the "true" Q^2 of the interaction assuming that the decay is dominated by $N^* \to p + h$ with h the daughter meson, and that the daughter hadron's momentum in the N^* center-of-mass frame is small (typically a few hundreds of MeV). In this case, the true Q^2 value can be either greater or less than the Q^2 values for when only inclusive electrons are detected at the same kinematics, see appendix A. Fortunately, the $t_{\rm int,Mo\&Tsai}$ value does not depend strongly on Q^2 and using an approximate value could be just fine, even that of the inclusive electrons' calculated using the hadron's momentum and scattering angle. On the other hand, as mentioned earlier, all electrons in the beam produce bremsstrahlung photons, while Mo&Tsai's equation applies only to electrons that are scattered. Therefore we consider Eq. (11) not optimal for estimating inclusive pion rates.

A better option for calculating $t_{\rm int}$ is to use the equivalent-photon approximation [3], with the spectrum of the photon given by [Eq.(8) of [3]]:

$$F_e^\gamma(E, y) = \frac{\alpha}{\pi}\ln\left(\frac{E}{m_e}\right)\frac{1 + (1-y)^2}{y} . \quad (12)$$

The function $F_e^\gamma(E, y)$ should replace the bremsstralung factor term $\frac{\alpha(K,K_0)}{K}$ in Eq. (2). Note that this spectrum is convoluted with the photoproduction cross section directly, without the use of Eq. (6), i.e., no extra factor $3/4$ should be added. Since Wiser code does the integration of σ_γ/E_γ with σ_γ the photoproduction cross section and $E_\gamma = yE$ the photon energy, the equivalent photon spectrum can be approximated by an input radiation length of

$$t_{\rm int,epa} \;=\; 2\frac{\alpha}{\pi}\ln\left(\frac{E}{m_e}\right), \tag{13}$$

where the term $\left[1+(1-y)^2\right]$ is replaced by 2 as an estimation of the upper limit and then taken out of the integration.

In short, the input RAD_LEN to the *Wiser* code should be either

$$t_{\rm ext+Mo\&Tsai} \;=\; \frac{4}{3}\frac{t_{\rm material}}{X_0} + \frac{\alpha}{\pi}\left[\ln\left(Q^2/m_e^2\right)-1\right], \tag{14}$$

or

$$t_{\rm ext+epa} \;=\; \frac{4}{3}\frac{t_{\rm material}}{X_0} + 2\frac{\alpha}{\pi}\ln\left(\frac{E}{m_e}\right), \tag{15}$$

depending on which equivalent photon radiator we use for the internal bremsstrahlung. For inclusive pion production, Eq. (15) is more relevant than Eq. (14). We have a slight preference for using Eq.(15) because all variables needed are well defined, while for Eq.(14) the value of Q^2 for the electron that leads to the hadron production cannot be easily calculated if only the hadron is detected.

We note the equivalent-photon approximation applies to real photon bremsstrahlung, which we expect to dominate over electro-production. This is important for parity violation electron scattering since photo-produced pions do not carry parity-violating asymmetries. A simple order-of-magnitude estimation shows that pion photoproduction to electroproduction ratio is simply the radiator thickness (a few %) times the photoproduction cross section (10 microbarn), divided by electroproduction cross section (typically 10-100nb). When the inclusive pion production is dominated by photoproduction, the asymmetry of pion samples is expected to be smaller than that of electrons', and the pion effect can be safely corrected as a dilution, provided the pion background contamination is below 10^{-3}.

1.2 From Proton to Nuclear Targets

The extension from the proton data to nuclear targets is done outside the *Wiser* code. Earlier version of some JLab code simply scaled the proton target yield by $A^{0.8}$. However, in this comparison we use the isospin symmetry method, where we assume that the π^- yield from the neutron is the same as the π^+ yield from the proton, etc. In this case, for a deuterium target the π^- cross section is the sum of the π^- and the π^+ cross sections from the *Wiser* code, and for a ^{3}He target the π^- cross section is the π^+ plus twice the π^- cross sections from the *Wiser* code, etc. We note that the isospin symmetry may work well for the DIS region, but not that well for the nucleon resonance region where the response of quarks to the beam is more coherent. It may also not work for low Q^2 below 1 (GeV/c)^2.

2 Comparison between Data and Calculation

2.1 Transversity experiment, polarized ^3He target

We use data from the HRS only. Rates from the BigBite are available but calculations of the BigBite acceptance is not easy to incorporate and could have large uncertainties. The transversity kinematics used (HRS only) are as follows:

- $E = 6.0$ GeV, $E' = 2.35$ GeV, $\theta = 16^0$;
- ^{3}He target, with target density assumed to be 10 amg, and a target length of 33 cm to match the cut used in the data analysis.
- Acceptance: momentum acceptance: $\pm 4.5\% E'$; angular acceptance: 5.7 msr. However, this may be overestimating the actual HRS by about 45%, given that the actual acceptance is more like $\pm 3.5\% E'$ and 4 msr (see page 216 of Xin Qian's Ph.D. thesis [5]).
- The drift distance from the target center to the first detector is 23.43 m in average, resulting in
  - Decay length for π : 173.8 m and number of particles remain: $e^{\frac{-23.43}{173.8}} = 87.4\%$.
  - Decay length for K : 17.34 m and number of particles remain: $e^{\frac{-23.43}{17.34}} = 25.89\%$.
- For electron rate, the code nmc_org.f is used.
- For pions and kaons, the *Wiser* code is used assuming isospin symmetry, i.e., the π^- rate from ^{3}He is calculated as the π^- rate plus twice the π^+ rate from the *Wiser* code.
- Radiation length, external from Ref. [6]:

  [0.001152(Be)+0.0001644(Air)+0.004281(Al)+0.001847(Glass)+0.000298(^{3}He)],

  multiplied by $4/3$ and 100 for converting to percentage.
- Radiation length, internal: both Mo&Tsai and the equivalent photon approximation were calculated. $rl = \frac{\alpha}{\pi} \ln(\frac{Q^2}{m_e^2})$ (Mo&Tsai) and $rl' = \frac{\alpha}{\pi} \ln(\frac{E_{beam}}{m_e})$ (equivalent-photon approximation).

Table 1: Comparison between calculation and data (in unit of #events/μC). The data are from Ref. [5]. The calculated rate for $\pi^\pm$ and K^- included decay. The radiator thickness values are: $t_{\rm ext} = 0.79\%$, $t_{\rm int,Mo\&Tsai} = rl = 3.54\%$, and $t_{\rm int,epa} = 2rl' = 4.36\%$. The electron rate was calculated to be 9 events/μC (without radiative corrections), compare to data 12.4 events/μC. The HRS acceptance used in this Table may be overestimating the actual values by 45%.

	Rate rl (Mo&Tsai)	Rate 2*rl' (e.p.a.)	Data
π^-	79.09	93.95	34
π^+	93.68	111.28	54.8
K^-	2.98	3.54	1.34

3 PVDIS experiment:

Conditions used for calculation are as follows:

- The external radiator is assumed to be dominated by the liquid deuterium target, which had a full thickness of 20 cm. The density used is 0.169 (g/cm^3). Additional material from the beam ling, the scattering chamber and the HRS entrance is assumed to be the same as for the 6 GeV A_1^n experiment as listed in Ref. [6], which adds a miniscule amount.

- The drift distance from the target center to the first detector is $23.43m$ in average.

- The HRS acceptance of $\Delta E' = \pm 4.5\% E'$ and $\Delta\Omega = 5.7$ msr were used.

- For electron rate, two codes are used: the nmc_org.f and F1F209.f.

- The pion rate was calculated using the *Wiser* code assuming isospin symmetry, i.e. the π^- rate for deuterium is the sum of π^- and π^+ from the *Wiser* code.

- An electron beam current 100 μA was used, consistent with the PVDIS running condition.

- Observed e and π rates are from Ref [7]. For π^- Ref. [7] gave the upper limit and we include the lower limit here.

Table 2: Comparison between calculation and data for e and π^- rates for the 6 GeV PVDIS experiment. The external radiator thickness is $t_{\text{ext}} = 1.79\%$. The internal radiator thicknesses are shown in the table. For the PVDIS experiment, two methods were used to extract the π^- rate from data: 1) rate of T1 trigger minus (rate of electron trigger divided by electron trigger efficiency); 2) rate of pion trigger divided by pion trigger efficiency, then remove electron contamination; which gave the upper and the lower limit of the rate range, respectively. We note that method #1 may include more background than π^- alone and thus could be an overestimation. The rates given in Ref. [7] were the average of the two methods, for most kinematics.

Kinematics				e rate [kHz]			π^- rate				
Kine#	E	E'	θ (°)				Mo&Tsai		e.p.a.		data
	(GeV)	(GeV)	(°)	F1F209	NMC	data	$t_{\text{int}}(\%)$	rate (kHz)	$t_{\text{int}}(\%)$	rate (kHz)	(kHz)
DIS I	6.067	3.66	12.9	269.1	271	210	3.55	186.6	4.36	214.7	(76,134)
DIS II	6.067	2.63	20.0	26.2	26.1	18	3.67	120.9	4.36	136.0	(53,66)
RES I	4.867	4.0	12.9	688	814	300	3.52	15.6	4.26	17.8	(22,72)
RES II	4.867	3.55	12.9	557	575	600	3.49	89.3	4.26	102.2	(82,125)
RES III	4.867	3.1	12.9	455	472	400	3.46	259.3	4.26	339.9	(137,162)
RES IV	6.067	3.66	15.0	125	128	80	3.62	67.9	4.36	77.1	(34,53)
RES V	6.067	3.66	14.0	128	184	130	3.59	110.8	4.36	126.6	(58,84)

4 Conclusion

For inclusive pion production, the equivalent radiator thickness that should be used as input to the *Wiser* code is given by Eq. 15. When comparing with 6 GeV DIS data (both ^{3}He and liquid deuterium targets), the *Wiser* code was found to overestimate the pion rate, but not as much as a factor of two as previously thought.

For 6 GeV PVDIS resonance kinematics, the *Wiser* code gave inconsistent results when comparing to data, and appears to be underestimating for RES I and II. However, the extraction of the pion rate for these kinematics were not reliable, in particular due to the small π/e ratio, < 0.25 for both RES I and II and the high overall rate (a few hundreds kHz). It's also possible that isospin symmetry does not work well for resonance regions. We can look for more data in the resonance region, preferrably from experiments that used HRS and thus had cleaner PID for the pions. On the other hand, a major concern is for pions produced at $E' < 1$ GeV, where data were not available from SLAD and thus were not included in the *Wiser* code. We hope to conduct test runs using the 11 GeV beam in Hall A in spring 2015, with the HRS set at below 1 GeV, to fill this gap.

A Pseudo-calculation of Q^2 for inclusive pion production

We assume that the pions are produced from nucleon resonance decays and are dominated by $N^* \to N\pi$ where N is either a proton or a neutron. The momentum of the daughter particle in the center-of-mass frame of the N^*, $|\vec{p}_f|$, is usually a few hundreds of MeV, and is quite small compare to the typical HRS momentum setting E'. We therefore ignore p_f and assume that both decay daughters move with the same 4-momentum as the N^*, which means $\vec{p}_N = \vec{p}_\pi = \vec{q}$ of the virtual photon. In this case, the kinematics of the decay product π can be uniquely determined from the electron's kinematics, and vice versa. We denote p_π and θ_π for the pion, and E' and θ for the scattered electron. It is straightforward to derive:

$$\tan\theta_\pi = \frac{E'\sin\theta}{E - E'\cos\theta} \tag{16}$$

$$p_\pi = |\vec{q}| = \sqrt{E^2 + E'^2 - 2EE'\cos\theta}\,. \tag{17}$$

If θ_π and p_π are known, E' can be determined by solving

$$E'^4 + E'^2\left(-2E^2 - 2|\vec{q}|^2\right) + \left[E^4 + 2|\vec{q}|^2 E^2(\sin^2\theta_\pi - \cos^2\theta_\pi) + |\vec{q}|^4\right] = 0 \tag{18}$$

with $|\vec{q}| = p_\pi$, and θ can be determined from

$$\cos\theta = \frac{E^2 + E'^2 - |\vec{q}|^2}{2EE'} \tag{19}$$

As one can see that the true Q^2 of the pion production may be either larger or smaller than the Q^2 of the inclusive electrons detected in the HRS, yet the pion background asymmetry was observed to be smaller than that of electrons. This indicates that the pion background was indeed dominated by that from photo- and not electro-productions.

References

[1] D. E. Wiser, UMI-77-19743. Ph.D. thesis, University of Wisconsin (1977). For experimental setup and data fit quality, see Chapter IV. For detailed kinematic coverage see tables listing the results.

[2] L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).

[3] M. S. Chen and P. M. Zerwas, Phys. Rev. D **12**, 187 (1975).

Table 3: Calculation of the true Q^2 for π^- inclusive production for the 6 GeV PVDIS experiment. Here we assume the π^- is produced only from $N^* \to N + \pi$ decays and ignore the $\vec{p}_f$, the decay daughter's momentum in the N^* center-of-mass frame. The Q^2 (DIS e') is the Q^2 value of the experiment.

Kine#	E (GeV)	p_π (GeV)	θ_π ($^\circ$)	[Q^2 (DIS e')] [(GeV/c)^2]	E' (GeV)	θ ($^\circ$)	Q^2 (true) [(GeV/c)^2]
DIS I	6.067	3.66	12.9	1.12	2.63	18.1	1.58
DIS II	6.067	2.63	20.0	1.93	3.71	14.1	1.34
RES I	4.867	4.0	12.9	0.98	1.32	42.7	3.4
RES II	4.867	3.55	12.9	0.87	1.61	29.4	2.02
RES III	4.867	3.1	12.9	0.76	1.97	20.6	1.22
RES IV	6.067	3.66	15.0	1.51	2.70	20.5	2.08
RES V	6.067	3.66	14.0	1.32	2.67	19.4	1.84

[4] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[5] Measurement of Single Spin Asymmetry in $n^\uparrow(e, e'\pi^\pm)X$ on Transversely Polarized ^{3}He, Xin Qian, Ph.D. thesis, Duke University (2010). For HRS acceptance cuts see 2D plots on page 216; for singles rates see Table 6.1 on page 265.

[6] http://hallaweb.jlab.org/physics/experiments/he3/A1n/ana/memo14/

[7] R. Subedi, D. Wang, K. Pan, X. Deng, R. Michaels, P. E. Reimer, A. Shahinyan and B. Wojtsekhowski *et al.*, Nucl. Instrum. Meth. A **724**, 90 (2013) [arXiv:1302.2854 [physics.ins-det]].